Test: Inference for One- and Two-way Tables and for Least-Squares Lines
Instructor's Version

<u>Free Response</u>: 2 questions, 52 points, 30 minutes

1. **Is gender associated with a person's environmental awareness? Two hundred randomly selected individuals were surveyed about their environmental awareness. Participants were asked about recycling, use of aerosol sprays, and various other environmental concerns. A point system was developed to score the individuals' responses. Once the points were tallied, individuals were broken up into categories of environmental awareness. Those with scores of 8 to 10 were considered to be very environmentally aware, scores of 4 to 6 moderately aware, and 3 or less not aware. The results are shown in the following table:**

	Very environmentally aware	Moderately environmentally aware	Not environmentally aware
Male	**30**	**21**	**45**
Female	**42**	**34**	**28**

Run an appropriate test of significance at $\alpha = 0.05$ to see if the data provide evidence that gender is associated with one's environmental awareness. (24 points)

Answer:

Hypotheses: H_o: There is no association between gender and one's environmental awareness.

H_α: There is an association between gender and one's environmental awareness.

Name of test: Chi-Square Test for Independence

Formula for test statistic: $x^2 = \sum \frac{(O-E)^2}{E}$

Conditions check: Sample is random. All expected counts must be > 5 and the value in each cell >1.

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Expected counts:

34.56	26.4	35.04
37.44	28.6	37.96

These counts can come from the calculator or can be found using the following formula for each of the 6 cells:

$$\frac{\text{row total} \bullet \text{column total}}{\text{total in sample}}$$

Statement: Since all expected counts are > 5, chi-squared test for independence is appropriate.

Mechanics: x^2 = 8.7256. This calculation must be shown for credit (see below).

p-value = 0.0127
df = 2

Calculations:
(30 – 34.56)²/34.56 + (21 – 26.4)²/26.4 + (45 – 35.04)²/35.04 + (42 - 37.44)²/37.44 + (34 – 28.6)²/28.6 + (28 – 37.96)²/37.96 = 0.6017 + 1.1045 + 2.8311 + 0.5554 + 1.0196 + 2.6133 = 8.7256

x^2cdf(8.7256,E99,2) = 0.0127

Decision: At $\alpha = 0.05$, reject H_o because *p*-value, 0.0127 < 0.05.

Conclusion: The sample data provide evidence that gender is associated with one's environmental awareness.

Scoring notes:
Award 2 points for a correct pair of hypotheses.
Award 2 points for correct test name.
Award 2 points for formula.
Award 2 points for conditions.
Award 4 points for showing the expected counts.
Award 2 points for statement regarding x^2 test for independence.
Award 6 points for correct mechanics with calculation shown (maximum 3 points if calculation not shown).
Award 2 points for correct decision with reason.
Award 2 points for correct conclusion in context. Do not award these points if there is no context.

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2. The following information was collected on 20 randomly chosen high school students, which compared their IQ score to musical aptitude.

Row	IQ Score	Mus.Apt.
1	95	30
2	106	37
3	110	35
4	104	28
5	98	17
6	114	45
7	123	33
8	100	18
9	98	24
10	88	18
11	102	30
12	105	30
13	111	32
14	106	33
15	110	38
16	143	42
17	106	40
18	98	20
19	104	23
20	97	25

Descriptive Statistics

Variable	N	Mean	Median	TrMean	StDev	SEMean
IQ Score	20	105.90	104.50	104.83	11.60	2.59
Mus.Apt.	20	29.90	30.00	29.78	8.27	1.85

Variable	Min	Max	Q1	Q3
IQ Score	88.00	143.00	98.00	110.00
Mus.Apt	17.00	45.00	23.25	36.50

Regression Analysis

The regression equation is

[ ] = [ ]

Predictor	Coef	Stdev	t-ratio	p
Constant	-22.26	12.94	-1.72	0.102
IQ Score	0.4925	0.1215	[ ]	[ ]

s = 6.143 R-sq = 47.7% R-sq(adj) = 44.8%

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A. What is the equation of the least squares regression line that would be used to predict musical aptitude from an IQ score? Be sure to define any variables used. (4 points)

Answer:
$\hat{y} = -22.26 + 0.4925x$, where x = IQ score and y = musical aptitude

Scoring notes:
Award 4 points for correct equation with variables defined.
Award 2 points for equation using x and y without stating what they represent.

B. Calculate and interpret a 95% confidence interval for the slope of the regression line. (6 points)

Answer:
Confidence interval from calculator: (0.23734, 0.74773)
By hand: $b \pm t^* SE_b = 0.4925 \pm 2.101(0.1215)$
$t^* = 2.101$ for $20 - 2 = 18$ degrees of freedom = 0.4925 ± 0.2553 = (0.2372, 0.7478)

Interpretation: It can be said with 95% confidence that the mean musical aptitude increases by between 0.24 and 0.75 points for each additional IQ point.

Scoring notes:
Award 2 points for a correct confidence interval.
Award 4 points for a correct interpretation.

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C. Is there a significant relationship between IQ score and musical aptitude? Give a statistical justification to support your answer. Assume conditions for regression inference are satisfied. Test at $\alpha = 0.05$. (12 points)

Answer:
Hypotheses: $H_o : \beta = 0$ and $H_\alpha : \beta \neq 0$

Test name: Linear regression *t*-test

Mechanics: *t* = 4.0535
t = 0.4925/0.1215 = 4.0535
p-value = 0.000746
1-tcdf(-4.0535,4.0535,18) = 0.000746
df = 20 – 2 = 18

Decision: At $\alpha = 0.05$, reject H_o because *p*-value 0.000743 < 0.05.

Conclusion: There is evidence of a significant relationship between IQ score and musical aptitude.

Scoring notes:
Award 2 points for a correct pair of hypotheses.
Award 2 points for correct test name.
Award 3 points for correct mechanics.
Award 2 points for correct decision with reason.
Award 3 points for correct conclusion with context.

D. From the computer printout, *s* = 6.143. Interpret this value in the context of the problem. (6 points)

Answer:
Any of the following interpretations should be awarded full credit.

s = 6.143 is the standard deviation of the residuals that measure a typical amount of variability in the vertical distances from the observed musical aptitude to the regression line.

The value *s* = 6.143 is a measure of variation in the musical aptitude for a given IQ score.

The value *s* = 6.143 is a typical prediction error for estimating musical aptitude from IQ score.

The value *s* = 6.143 is the estimated standard deviation of the differences between the observed values for musical aptitude and the values predicted by the regression line.

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